# TMUA EUCLID

## r2drew2 edition

PAPER 1
75 MIN. • NO CALCULATOR


Written by TMUA Euclid

**1.** The roots of

$$x^2 + px + q = 0$$

are $a$ and $b$.
The roots of

$$x^2 + qx + p = 0$$

are $a + 1$ and $b + 1$.
What is $p + q$?

**R** $-6$

**2** $-5$

**D** $-4$

**r** $-3$

**E** $-2$

**W** $-1$

**2.** An integer $n \geq 3$ has the following property:
In the expansion of

$$(1+x)^n(1-x),$$

the coefficient of $x^2$ is equal to the coefficient of $x^3$.
What is $n$?

**R** $5$

**2** $6$

**D** $7$

**r** $8$

**E** $9$

**W** $10$

**3.** The equation

$$9^x - 10 \cdot 6^x + 9 \cdot 4^x = 0$$

has solutions $p$ and $q$.
What is

$$\left(\frac{3}{2}\right)^{p+q}?$$

**R** $1$

**2** $9$

**D** $10$

**r** $81$

**E** $3$

**W** $27$

**4.** The function $f$ is defined by

$$f(x) = x^2 - 6x + 7.$$

Distinct real numbers $a$ and $b$ satisfy

$$f(a) = f(b) \qquad \text{and} \qquad f(a+1) = f(b-2).$$

What is $ab$?

**R** $\frac{9}{4}$

**2** $\frac{9}{2}$

**D** $6$

**r** $\frac{27}{4}$

**E** $9$

**W** $12$

**5.** An integer $n > 3$ satisfies

$$n^2 - 9 = 2^k,$$

where $k$ is a non-negative integer.
What is $k$?

**R** $3$

**2** $4$

**D** $5$

**r** $6$

**E** $7$

**W** $8$

**6.** The circles

$$x^2 + y^2 = 25$$

and

$$(x - 8)^2 + y^2 = r^2$$

intersect in a common chord of length $8$.
There are two possible values of $r^2$.
What is their sum?

**R** $162$

**2** $170$

**D** $178$

**r** $186$

**E** $194$

**W** $202$

**7.** When the polynomial $P(x)$ is divided by $(x-1)^2$, the remainder is $4x-1$.
What is the remainder when

$$P(x^2-3x+3)$$

is divided by $(x-2)^2$?

**R** $2x-1$

**2** $3x-3$

**D** $6x-9$

**r** $5x-7$

**E** $4x-5$

**W** $7x-11$

**8.** For real $x$, define

$$F(x) = |x^2 - 5x| + |x^2 - 5x + 4|.$$

The set of values of $x$ for which $F(x)$ is least consists of two closed intervals. What is the sum of the lengths of these intervals?

**R** $1$

**2** $2$

**D** $\frac{5}{2}$

**r** $3$

**E** $\frac{3}{2}$

**W** $4$

**9.** In triangle $ABC$,

$$AB = 12, \qquad AC = 7, \qquad \angle ABC = 30^\circ.$$

There are two non-congruent triangles satisfying these conditions. Their areas are $K_1$ and $K_2$. What is $K_1 K_2$?

**R** $720$

**2** $810$

**D** $855$

**r** $900$

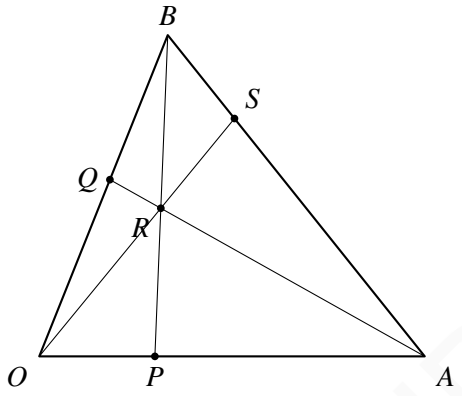
**E** $1080$

**W** $960$

**10.** In triangle $OAB$, the point $P$ lies on $OA$ and the point $Q$ lies on $OB$, where

$$OP : PA = 2 : 3, \qquad OQ : QB = 3 : 4.$$

The lines $AQ$ and $BP$ meet at $R$. The line $OR$ meets $AB$ at $S$.




*[diagram not to scale]*

What is $AS : SB$?

**R** $4 : 3$

**2** $9 : 8$

**D** $8 : 9$

**r** $3 : 2$

**E** $2 : 3$

**W** $1 : 1$

**11.** The equation

$$\log_2(x^2 - 5x + 8) + \log_2(8 - x^2 + 5x) = 4$$

has two real solutions $p$ and $q$.
What is $pq$?

**R** $-8\sqrt{3}$

**2** $-4\sqrt{3}$

**D** $-2\sqrt{3}$

**r** $2\sqrt{3}$

**E** $4\sqrt{3}$

**W** $8\sqrt{3}$

**12.** Evaluate

$$\int_0^1 \frac{x^6 + x^3 + 1}{x^3 + x^{3/2} + 1}\, dx.$$

**R** $\frac{17}{20}$

**2** $\frac{7}{8}$

**D** $\frac{11}{12}$

**r** $\frac{5}{6}$

**E** $\frac{14}{15}$

**W** $1$

**13.** For $x > 0$, define

$$F(x) = \frac{x^4 + 4x^2 + 1}{x^4 + 2x^3 + 3x^2 - 2x + 1}.$$

What is the least possible value of $F(x)$?

**R** $\frac{10}{13}$

**2** $\frac{4}{5}$

**D** $1$

**r** $2$

**E** $\frac{3}{4}$

**W** $\frac{6}{5}$

**14.** A point $P$ lies inside triangle $ABC$.
Through $P$, a line is drawn parallel to $BC$. This line cuts off a smaller triangle at $A$ with area $4$.
Similarly, the line through $P$ parallel to $CA$ cuts off a smaller triangle at $B$ with area $9$, and the line through $P$ parallel to $AB$ cuts off a smaller triangle at $C$ with area $16$.
Each of the three lines is considered separately.
What is the area of triangle $ABC$?

**R** $\frac{29}{2}$

**2** $\frac{81}{4}$

**D** $29$

**r** $\frac{81}{2}$

**E** $81$

**W** $36$

**15.** What is the sum of all values of $\theta$ satisfying

$$0 \leq \theta < 2\pi$$

and

$$|\sin\theta| + |\cos\theta| = \frac{1+\sqrt{3}}{2}\,?$$

**R** $4\pi$

**2** $6\pi$

**D** $8\pi$

**r** $10\pi$

**E** $12\pi$

**W** $14\pi$

**16.** At three distinct points $A$, $B$ and $C$ on the parabola

$$y = x^2,$$

the normals to the parabola all meet at the same point $P$.
The point $A$ has coordinates

$$A = (1, 1),$$

and the sum of the $y$-coordinates of $A$, $B$ and $C$ is $14$.
What is the $x$-coordinate of $P$?

**R** $-24$

**2** $-12$

**D** $-6$

**r** $6$

**E** $12$

**W** $24$

**17.** A line is called a **GP-secant** of a curve if it meets the curve at three distinct points whose $x$-coordinates are positive and form a geometric progression.
A GP-secant of

$$y = x^3 - 12x^2 + 7x$$

passes through $(1, -9)$.
What is the gradient of the GP-secant?

**R** $-13$

**2** $-16$

**D** $-17$

**r** $-18$

**E** $-25$

**W** $-24$

**18.** For a real number $a$, consider the graphs

$$y = |x^2 - 1|$$

and

$$y = |(x - a)^2 - 1|.$$

The graphs intersect at exactly three distinct points.
Which of the following describes all the possible values of $a$?

**R** $a = 0$ only

**2** $-2 < a < 0$ or $0 < a < 2$

**D** $-2 \leq a \leq 2$

**r** $a < -2$ or $a > 2$

**E** $a \neq 0$

**W** $a = \pm 2$ only

**19.** A rectangle $ABCD$ has area $3$.
The points $P$, $Q$ and $R$ lie on the sides $BC$, $CD$ and $DA$, respectively.
Among all possible rectangles $ABCD$ of area $3$, and all possible positions of $P$, $Q$ and $R$, what is the least possible value of

$$AP + PQ + QR + RB\,?$$

**R** $2\sqrt{6}$

**2** $3\sqrt{3}$

**D** $4\sqrt{2}$

**r** $6$

**E** $3\sqrt{5}$

**W** $4\sqrt{3}$

**20.** Let

$$f(x) = x^2 - 2.$$

Define $f_1(x) = f(x)$ and, for $n \geq 1$,

$$f_{n+1}(x) = f\big(f_n(x)\big).$$

How many distinct real solutions does the equation

$$f_8(x) = 2$$

have?

**R** $65$

**2** $127$

**D** $128$

**r** $129$

**E** $256$

**W** $255$